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Design of Fuzzy Logic Guidance Law Against High-Speed Target

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An advanced guidance law is developed against very high-speed targets. Preliminary studies have shown that the aspect angle of the interceptor at lock-on near 180 deg is a fundamental requirement for achieving small miss distance against a very high-speed target. To meet this requirement, a fuzzy guidance law in midcourse phase that is more similar to human decision making is designed. In terminal phase, a proportional and derivative-type fuzzy terminal guidance law is explored. It is shown that the integrated guidance scheme offers a near head-on homing condition before the missile enters terminal phase and provides better final results (smaller miss distance and wider defensible volume) than the conventional guidance law. A complete simulation study is performed to show the effects of the proposed design.

Nomenclature

C_D = drag coefficient
C_{D0} = zero drag coefficient
C_L = lift coefficient
$C_{L\alpha}$ = $\partial C_L/\partial\alpha$
D = drag
h = vertical coordinate
L = lift
m = mass
Q = dynamic pressure
S_{tref} = reference area of ballistic target
s = reference area
T = thrust
v = missile speed
v_t = target speed
W = target weight
x = horizontal coordinate
α = angle of attack
β = ballistic coefficient
γ = flight-path angle
γ_t = reentry angle of target
δ = velocity angle error
θ = inertial line-of-sight angle
μ = induced drag coefficient
ρ = atmospheric density
σ = heading error angle

I. Introduction

CURRENTLY, most tactical defense missiles are designed mainly for use against aircraft. Putting aside hardware limitations, with no additional geometry constraint imposed on the missile flight trajectory, the resulting navigation system can only pursue targets with a lower speed than the defense missile. It has been investigated and is reflected in the literature that the most effective way for a lower speed interceptor to successfully engage an incoming target with very high speed is to limit the aspect angle between the missile and target flight path to within 180 deg plus or minus few degrees.[1,2] Such a geometric arrangement will minimize the lateral acceleration level for effectively engaging hypersonic targets.

Theoretically, the missile–target dynamics are highly nonlinear, partly because the equations of motion are best described in an inertial system, whereas aerodynamic forces and moments are represented in the missile and target body axis system. In addition, unmodeled dynamics or parametric perturbations usually remain in the plant modeling procedure. Because of complexity of the nonlinear guidance design problem, prior approximations or simplifications were usually required before deriving the analytical guidance gains.[3–5] Therefore, one typically does not know exactly what the true missile model is, and the missile behavior may change in unpredictable ways. Consequently, optimality of the resulting design cannot be ensured.

Very often, no preferred mathematical model is presumed in its problem formulation, and information is presented in a descriptive manner. Recently developed neural network algorithms and fuzzy logic theory serve as possible approaches, solving for the highly nonlinear flight control problems.[6–8] However, a very limited number of papers have addressed the issue of fuzzy missile guidance design.[9,10] The use of fuzzy logic control is motivated here by the need to deal the nonlinear flight control and performance robustness problems. Fuzzy control based on fuzzy logic provides a new design paradigm such that a controller can be designed for complex, ill-defined processes without quantitative data regarding the input–output relations, which are otherwise required by conventional approaches.[11]

Speed plays an important role in determining missile aerodynamic maneuverability. Decreasing the missile speed significantly decreases the missile maneuverability. The interceptor acceleration capability increases with decreasing altitude, whereas the target deceleration capability also increases with decreasing altitude. From the interceptor viewpoint, the ideal intercept should take place at very low altitude, where the interceptor has enormous capability and a considerable acceleration advantage over the target. However, practical considerations may require the interceptor to engage the ballistic target at much higher altitudes. For a given altitude and missile configuration, there is a minimum speed requirement such that the missile can effectively engage a responsive target. Based on this requirement and the preceding investigation, a fuzzy midcourse guidance law is developed to increase the terminal speed by human expertise and to bring the target aspect angle to as close to 180 deg at hand over as is possible. In the presented design, the trajectory shaping is determined by the position of the predicted lock-on point relative to the current and final flight-path angles. In terminal phase, a fuzzy terminal guidance is designed by using the proportional and derivative-type (PD-type) fuzzy control methodology. This guidance configuration accelerates the tracking response and avoids overshooting the state response. Complete simulation results show that the proposed guidance scheme offers satisfactory

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miss distance and wider defensible volumes than the conventional guidance design and, in addition, possesses a robustness feature to plant variations.

II. Guidance Law Design Scheme

Two-dimensional translation equations of motion are used to compute the trajectory of the guided missile. The missile is modeled as a point mass, and the equations of motion are[1,2]

$$\dot{x} = v\cos\gamma \tag{1a}$$

$$\dot{h} = v\sin\gamma \tag{1b}$$

$$\dot{v} = (T\cos\alpha - D - mg\sin\gamma)/m \tag{1c}$$

$$\dot{\gamma} = (L - mg\cos\gamma + T\sin\alpha)/mv \tag{1d}$$

where the aerodynamic forces D and L are

$$L = \tfrac{1}{2}\rho v^2 s C_L, \qquad D = \tfrac{1}{2}\rho v^2 s C_D$$

with $C_L = C_{L\alpha}(\alpha - \alpha_0)$ and $C_D = C_{D0} + \mu C_L^2$. In the preceding equations, the angle of attack α is used as the control variable. The thrust T and mass m are predefined functions of time. The initial and terminal constraints on the states are

$$x(0) = 0, \qquad h(0) = h_0, \qquad v(0) = v_0$$

$$\gamma(0) = \gamma_0, \qquad x(t_f) = x_f, \qquad h(t_f) = h_f$$

where t_f is the final flight time.

It is well known that the simplest midcourse guidance is the explicit guidance law.[12] The guidance algorithm has the capability to guide the missile to a desired point in the air while controlling the approach angle and minimizing an appropriate cost function.[13] The guidance gains of the explicit guidance law are usually selected to shape the trajectory for the desired attributes.[14,15] The inertial acceleration command generated by the guidance algorithm is in the form

$$\boldsymbol{a} = (K_1/t_{go})(\boldsymbol{v}_f - \boldsymbol{v}_0) + (K_2/t_{go}^2)(\boldsymbol{r}_f - \boldsymbol{r}_0 - \boldsymbol{v}_0 t_{go}) \tag{2}$$

where $t_{go} = t_f - t$ is the time to go, $\boldsymbol{r} = [x \;\; h]^T$, $\boldsymbol{v} = [v_x \;\; v_h]^T$, and $\boldsymbol{a} = [a_x \;\; a_h]^T$ are, respectively, the position, velocity, and acceleration vectors. Traditionally, the gains K_1 and K_2 are obtained by minimizing

$$J = \int_{t_0}^{t_f} \boldsymbol{a}\cdot\boldsymbol{a}\,\mathrm{d}t \tag{3}$$

subject to

$$\dot{\boldsymbol{r}} = \boldsymbol{v}, \qquad \boldsymbol{r}(t_0) = \boldsymbol{r}_0, \qquad \dot{\boldsymbol{v}} = \boldsymbol{a}, \qquad \boldsymbol{v}(t_0) = \boldsymbol{v}_0$$

and the given final constraints

$$\boldsymbol{r}(t_f) = \boldsymbol{r}_f, \qquad \boldsymbol{v}(t_f) = \boldsymbol{v}_f$$

It can be shown that the gains $K_1 = -2$ and $K_2 = 6$ are one of the solutions that satisfy the boundary conditions.[5,12] However, they do not possess the optimal sense.

To reduce the effect of uncontrolled axial acceleration command, the normal acceleration is first transformed into the seeker coordinate with reference to the simplified two-dimensional intercept geometry as shown in Fig. 1. Using the approximation $R \cong v t_{go}$ with R being the slant range between $\boldsymbol{r}$ and $\boldsymbol{r}_f$, the normal acceleration can be represented as

$$a_n = K_1 a_v + K_2 a_p \tag{4}$$

where

$$a_v = -(v/t_{go})\sin\delta = -(v^2/R)\sin\delta$$

$$a_p = (R/t_{go}^2)\sin\sigma = (v^2/R)\sin\sigma$$

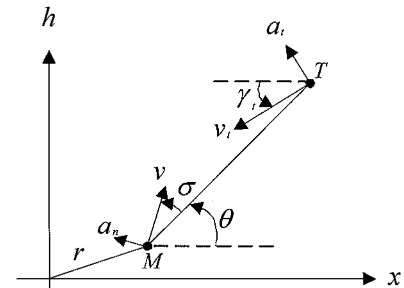

Fig. 1 Intercept geometry.

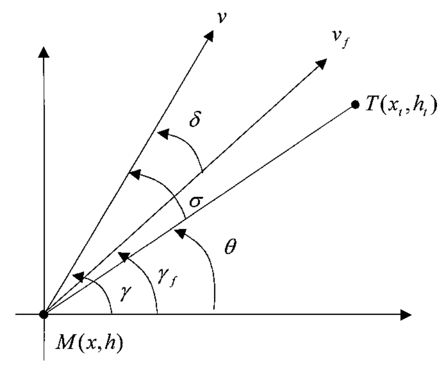

Fig. 2 Definitions of σ and δ.

with the predicted velocity error angle between the present and final vectors

$$\delta = \gamma_f - \gamma \tag{5}$$

and the heading error angle

$$\sigma = \gamma - \theta \tag{6}$$

in which the missile–target line-of-sight (LOS) angle is θ. Definitions of δ and σ are shown in Fig. 2. The first term of the right-hand side of Eq. (4) is a trajectory shaping term that commands the missile toward a prescribed approach angle. The second term is a proportional navigation term that generates acceleration to reduce the predicted miss distance.

The control variable α in the dynamic equation (1) is related to the lateral acceleration command a_n by

$$a_n = (1/2m)\rho v^2 s C_{L\alpha}\alpha \tag{7}$$

The explicit guidance expression (4) offers a way of designing the fuzzy logic-based guidance law. It is seen that the plant described by Eq. (1) and the guidance law (4) are nonlinear with respect to the state and control variables. Furthermore, the lift coefficient is nonlinear with respect to α. Therefore, calculation for the actual acceleration command a_n from Eq. (7) with respect to α is nonlinear as well. The rule-based representation of a fuzzy logic controller (FLC) does not include any dynamics, and the computational structure of an FLC consisting of fuzzification, inference, and defuzzification is highly nonlinear. These factors make an FLC a natural nonlinear static transfer element like a static controller. This specific feature is especially appropriate for the preceding guidance design. Based on this observation, let the fuzzy midcourse guidance law have the form

$$\alpha_{fm} = f(\delta, \sigma) \tag{8}$$

where $f(\cdot,\cdot)$ is the input–output mapping of a fuzzy logic system. The variable δ is contributed by the velocity error vector that specifies the predicted terminal speed relative to the current velocity. The variable σ is contributed by the position error vector that specifies the terminal position relative to the current position.

When the missile approaches homing at the end of midcourse phase, trajectory shaping becomes less important whereas minimizing the position error for good accuracy becomes more important. As the missile enters the terminal phase, the effect resulting from the velocity error is ignored. It would be better to apply the position error message σ to design the terminal guidance law such that the aspect angle between the missile and target flight path is close to 180 deg. For the fuzzy terminal guidance law, it is natural to select the heading error angle σ and change of heading error angle $\dot{\sigma}$ as the antecedent variables like a PD-type controller, that is,

$$\alpha_{ft} = f(\sigma, \dot{\sigma}) \tag{9}$$

Note that for antiaircraft missiles, the terminal guidance law is usually the proportional navigation guidance. The guidance law, described later, is well known to effectively engage evasive targets:

$$a_n = NV_c\dot{\theta}$$

where N is the proportional navigation gain and V_c is the closing velocity. If the mission objective is to engage an evasive target, the following fuzzy guidance law is suggested:

$$\alpha_{ft} = f(\dot{\theta}, \ddot{\theta})$$

Because the control variable α has been determined, the lateral acceleration command to be fed into the plant can be obtained by substituting the control variable into Eq. (7).

III. Counter Attack Strategy

An important guideline for a successful engagement is that the missile and the target are at near head-on geometry. This will minimize the LOS rotational rates and avoid saturating the command signals at terminal phase. The guidance scheme proposed here possesses three phases: midcourse phase, shaping phase, and terminal phase. A fuzzy midcourse guidance law first navigates the missile. As the seeker acquires the incoming target, the missile enters terminal phase, and a fuzzy terminal guidance law is activated. Between the midcourse and terminal phases is the shaping phase. When the missile is entering this phase, shaping guidance activates such that the guidance commands issued from the midcourse, and terminal phases are transferred smoothly. Figure 3 illustrates the geometric relation between a defense missile and an incoming target.

To drive the missile attaining the near head-on geometry before the homing guidance phase, we define the range to go

$$R_p = R + R_{lock} \tag{10}$$

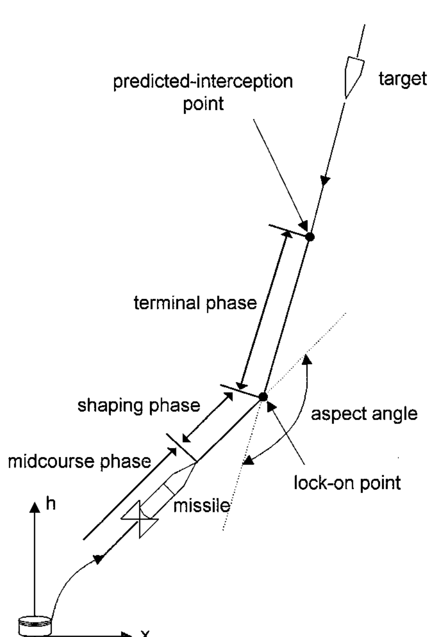

Fig. 3 Engagement geometry.

where R_{lock} is the seeker lock-on range that is assumed to be constant. In the actual implementation of the guidance law, we first estimate the required time to go for the target attaining the predicted lock-on point:

$$t_{gop} = -\frac{R_p}{\dot{R}_p} = \frac{R_p R}{R_{mtx}v_{rx} + R_{mth}v_{rh}} \tag{11}$$

where the relative range and velocity in x and h coordinates are

$$R_{mtx} = x_t - x, \qquad v_{rx} = v_x - v_{tx}$$

$$R_{mth} = h_t - h, \qquad v_{rh} = v_h - v_{th}$$

with

$$v_x = v\cos\gamma, \qquad v_h = v\sin\gamma$$

and

$$R = \sqrt{R_{mtx}^2 + R_{mth}^2} \tag{12}$$

Based on t_{gop}, the predicted lock-on point can be estimated by

$$x_f = x_t + v_{tx}t_{gop}, \qquad h_f = h_t + v_{th}t_{gop} \tag{13}$$

The predicted LOS angle with respect to the lock-on point is given by

$$\hat{\theta} = \tan^{-1}[(\hat{h}_t - h)/(\hat{x}_t - x)] \tag{14}$$

where

$$\hat{x}_t = x_t + v_{tx}\tilde{t}, \qquad \hat{h}_t = h_t + v_{th}\tilde{t}$$

with

$$\tilde{t} = -\frac{R_{lock}}{\dot{R}_p} = \frac{R_{lock}R}{R_{mtx}v_{rx} + R_{mth}v_{rh}}$$

To attain near head-on geometry, we specify the desired flight-path angle as

$$\hat{\gamma}_f = \tan^{-1}[(h_f - h)/(x_f - x)] \tag{15}$$

In the guidance law design scheme, the LOS angle θ and final flight-path angle γ_f in Eqs. (5) and (6) are, respectively, replaced by $\hat{\theta}$ and $\hat{\gamma}_f$.

In the preceding equations, target position and velocity information in the Cartesian inertial frame are the uplink data obtained from a ground-based radar or satellite. Missile position and velocity information in the Cartesian inertial frame are obtained from an inertial reference unit. A sophisticated estimation technique can be accompanied to treat these raw data and, hence, obtain better estimates of $\hat{\gamma}_f$.

IV. Fuzzy Rule-Based Guidance Law

Designing the fuzzy guidance rule is inspired by the previous research,[8] which proposes a conventional optimal guidance law to acquire better counterattack conditions and lower energy consumption.

A. Midcourse Phase: Fuzzy Logic-Based Midcourse Guidance (FLMG)

The sequence of operations in a fuzzy system can be described by three phases, namely, fuzzification, inference, and defuzzification. Design steps for the fuzzy guidance law will be introduced in the following sections.

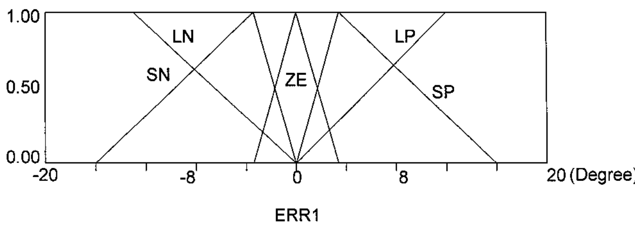

Fig. 4 Membership function of ERR1.

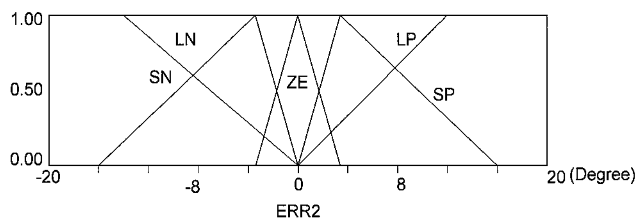

Fig. 5 Membership function of ERR2.

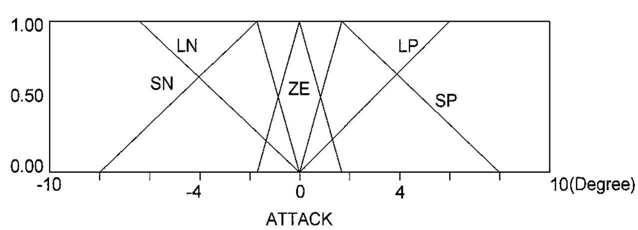

Fig. 6 Membership function of ATTACK.

1. Fuzzification

The input and output variables of a fuzzy system are the linguistic variables because they take linguistic values. The input linguistic variables of fuzzy logic-based midcourse guidance (FLMG) are ERR1 (σ) and ERR2 (δ), and the output variable is ATTACK (α). The universe of discourse of the linguistic variable ERR1 is supposed to be $[-20, 20]$ deg, ERR2 is $[-20, 20]$ deg, and ATTACK is $[-10, 10]$ deg.

The linguistic values taken by these variables are expressed by linguistic sets. Each of the linguistic variables is assumed to take five linguistic sets defined as large negative (LN), large positive (LP), small negative (SN), small positive (SP), and zero (ZE). The linguistic sets are described by their membership functions as shown in Figs. 4–6. To simplify the computation in the actual operation, triangular membership functions are suggested. It has been found that using complex forms of membership functions, such as bell-shaped functions, cannot bring any advantage over the triangular ones. For a nonevasive target, tracking errors in midcourse phase are generally larger than those in terminal phase. To avoid overshooting the control command, low distinguishable fuzzy sets (compared with those used in terminal guidance) are suggested. Fuzzification refers to the process of determining the degree of membership of crisp input data among the variable's membership function set.

2. Rule Base

The rule base contains a collection of rules and forms an integral part of the total knowledge embedded in the guidance computer. In midcourse phase, it is hoped that the missile reaches the lock-on point with as high a speed as possible. Therefore, the velocity error angle is considered more important than the heading error angle. In the rule base design, the former is placed with higher weight than the latter. Therefore, it could be expected that the velocity error angle would converge more quickly than the heading error angle during midcourse phase. As with most fuzzy logic-based control designs, a great deal of experiments will be needed to confirm the prototype of guidance rules. A set of 25 guidance rules has been refined to meet our purpose. The complete rule table is given in Fig. 7. For illustration, some sample rules are given as follows:

α_{fm}		σ				
		LN	SN	ZE	SP	LP
δ	LN	LN	LN	LN	LN	LN
	SN	SN	SN	SN	SN	LN
	ZE	SP	SP	ZE	SN	SN
	SP	LP	SP	SP	SP	SP
	LP	LP	LP	LP	LP	LP

: group 1
: group 2
: group 3
: group 4
: group 5

Fig. 7 Rule table of fuzzy midcourse guidance.

If ERR1 is LN and ERR2 is LP then ATTACK is LP
If ERR1 is SP and ERR2 is SN then ATTACK is SN

The set of rules can be divided in the following five groups:

1) In group 1, δ is close to zero. This means that the current flight-path angle γ is consistent with the desired flight path angle γ_f. The control action is thus intended to correct the heading error angle σ. However, the amount is small or close to zero so that the current δ will not be altered.

2) For group 2, γ is significantly greater than γ_f. The control action is intended to significantly reverse this trend. The heading error angle σ is irrelevant.

3) For group 3, γ is slightly greater than γ_f. For the situations of $\sigma \leq 0$, the control action is impossible to compensate for both σ and δ at the same time. The control action is dedicated to compensate for δ because it is considered to be more important. For $\sigma > 0$, the control action is intended to compensate for σ and δ, and its amount is the average of the two factors.

4) Control actions of groups 4 and 5 are, respectively, the opposite of groups 3 and 2.

3. Rule Evaluation (Inference)

We use the max–min (Mamdani–Assilian type) inference[16] to generate the best possible conclusions. In this inference mechanism, the min and max operations are, respectively, used for the AND and OR operations. This type of inference is computationally easy and effective; thus, it is appropriate for real-time control applications. The fuzzified inputs fire according to each rule individually. The clipped membership functions of the individual rule are then merged to produce one final fuzzy set. The max operation is used to merge overlapping regions.

4. Defuzzification

The outputs of the linguistic rules are fuzzy, but the guidance command must be crisp. Therefore, the outputs of the linguistic rules must be defuzzified before feeding into the plant. The crisp control action is calculated here using the c.g. center-of-area (COA) defuzzification procedure. The criterion provides defuzzified output with better continuity. For a plant that is sensitive to the command quality such as missile autopilot, this criterion will be more appropriate than other defuzzification methods.

B. Shaping Phase

When the relative range of missile and target is smaller than the appropriate shaping range (prior to seeker lock-on), the shaping guidance law activates. In this phase, the guidance commands issued from midcourse phase are transferred smoothly to terminal phase. FLMG remains to be used in this phase. When a missile enters the shaping phase, a common approach is to keep the gain K_1 and then linearly reduce it to zero with respect to the time interval of the shaping phase. A fuzzy logic-based guidance law does not use any explicit guidance gain. Therefore, an equivalent approach is to keep the input value ERR2 of FLMG and then linearly reduce it to zero. Other operations in this phase remain as invariant as those in midcourse phase.

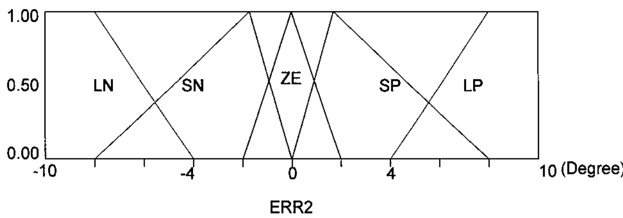

Fig. 8 Membership function of ERR2.

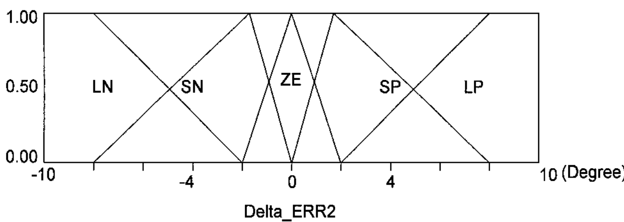

Fig. 9 Membership function of Delta_ERR2.

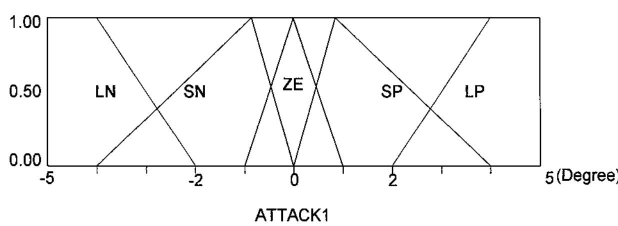

Fig. 10 Membership function of ATTACK1.

α_{ft}		$\dot{\sigma}$				
		LN	SN	ZE	SP	LP
σ	LN	LP	LP	LP	LP	LP
	SN	LP	SP	SP	ZE	ZE
	ZE	LP	SP	ZE	SN	LN
	SP	ZE	ZE	SN	SN	LN
	LP	LN	LN	LN	LN	LN

: group 1
: group 2
: group 3
: group 4
: group 5

Fig. 11 Rule table of fuzzy terminal guidance.

C. Terminal Phase: Fuzzy Logic-Based Terminal Guidance (FLTG)

As the seeker acquires the incoming target, the terminal guidance activates. In the terminal phase, the position error dominates the final miss distance. The influence resulting from the velocity error becomes less important. The navigation term contributed by the heading error angle generates an effective acceleration command to reduce the final miss distance. As described in the preceding sections, the fuzzy logic terminal guidance (FLTG) uses the heading error angle σ and the change of the heading error angle $\dot{\sigma}$ as the input linguistic variables. We denote the input variables as ERR2 (σ) and Delta_ERR2 ($\dot{\sigma}$), and denote the output variable as ATTACK1 (α).

The universe of discourse for the input linguistic variable ERR2 is supposed to be [−10, 10] deg, Delta_ERR2 is [−10, 10] deg, ATTACK1 is [−5, 5] deg. For these variables, five linguistic sets (LN, SN, ZE, SP, LP) have been considered and triangular membership functions have been assumed. The corresponding membership functions are shown in Figs. 8–10. In terminal phase, the aspect angle is supposed to near 180 deg. For a nonevasive target, the tracking error should be less than that in midcourse guidance. Thus, high distinguishable fuzzy sets can be used, to increase the control sensitivity. A rule table analogous to the standard PD-type rule table is constructed in Fig. 11. For illustration, some sample rules are given here:

If ERR2 is SN and Delta_ERR2 is SN then ATTACK1 is SP
If ERR2 is SP and Delta_ERR2 is SP then ATTACK1 is SN

Referring to Fig. 11, the set of rules is divided in the following five groups:

1) In group 1, both σ and $\dot{\sigma}$ are small or zero. This means that the current flight-path angle γ is close to the LOS angle θ. The amount of control action is also small or close to zero and is intended to correct small deviations from θ. Therefore, the rules are related to the steady-state behavior of the missile.

2) For group 2, γ is either significantly greater than θ or close to it. At the same time, $\dot{\sigma} > 0$ means that the missile is moving away from θ. The control action is intended to significantly reverse this trend.

3) For group 3, γ is smaller than θ. At the same time, $\dot{\sigma} > 0$ means that the missile is moving toward θ, that is, it is self-correcting to the desired direction. The control action is intended to speed up the approach to θ. When σ is small negative, the control action is set to zero to avoid overshooting the response.

4) Control actions of groups 4 and 5 are, respectively, the opposite of groups 2 and 3.

The rule base is shown to provide excellent control strategy for speeding up transient response and avoid too much overshoot. The rule evaluation in terminal guidance is still the max–min inference. To calculate the crisp control action from the output of the linguistic rules, a COA defuzzification procedure has been used.

Fuzzy control systems are essentially nonlinear systems. Therefore, it is difficult to obtain general results on the analysis and design of FLMG and FLTG. In addition, knowledge of the missile's aerodynamics is normally poor. Therefore, robustness of the guidance and control systems must be evaluated to guarantee stability and performance when variations in aerodynamic coefficients we present.

V. Case Study

A. Ballistic Target Model

When a ballistic target reenters the atmosphere after having traveled a long distance, its speed will be very high and remaining time to ground impact will be relatively short. The small displacement distance traveled by ballistic targets after they reenter the atmosphere enables us to accurately model these threats using the flat-Earth, constant gravity approximation as was done in modeling tactical interceptors.[17]

Based on earlier assumptions, suppose that only drag and gravity acting on the endoatmospheric ballistic target are considered. Let the target have velocity v_t and an initial reentry angle γ_t. The downrange of the target is x_t, and the altitude is h_t. Note that the drag force F_{drag} acts in a direction opposite to the velocity vector, and gravity g always acts downward. Therefore, if the effect of drag is greater than that of gravity, the target will decelerate. The target reentry angle can be computed using the two inertial components of the target velocity:

$$\gamma_t = \tan^{-1}(-v_{th}/v_{tx}) \tag{16}$$

The acceleration components of the ballistic target in the inertial downrange and altitude directions can either be expressed in term of the target's weight W_t, reference area S_{tref}, zero lift drag C_{tD0}, and gravity g, or, more simply, in terms of the ballistic coefficient β according to the following equations[17]:

$$\frac{\mathrm{d}v_{tx}}{\mathrm{d}t} = \frac{-F_{\text{drag}}}{m_t}\cos\gamma_t = \frac{-Qg}{\beta}\cos\gamma_t$$
$$\frac{\mathrm{d}v_{th}}{\mathrm{d}t} = \frac{F_{\text{drag}}}{m_t}\sin\gamma_t - g = \frac{Qg}{\beta}\sin\gamma_t - g \tag{17}$$

where m_t is the target mass and

$$\beta = W_t / C_{tD0} S_{\text{tref}} \tag{18}$$

The air density ρ is measured in kg/m^3 and is approximated as

$$\rho = 0.12492(1 - 0.000022557 h_t)^{4.2561} g \tag{19}$$

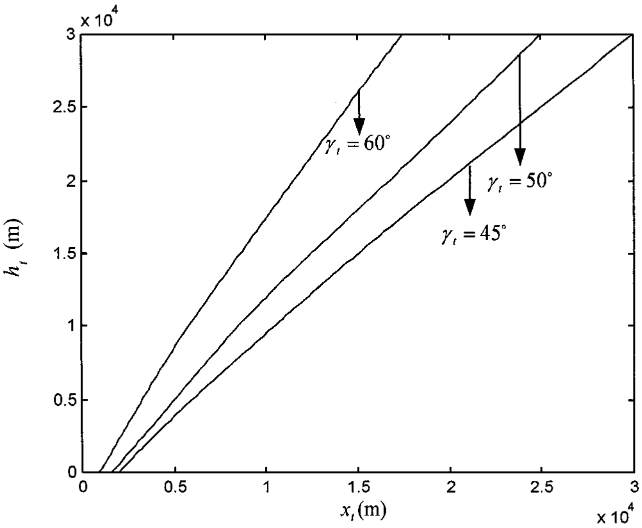

Fig. 12a Target flight trajectories.

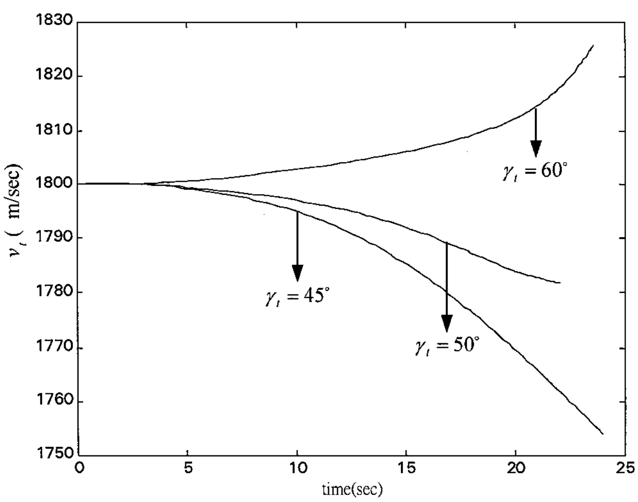

Fig. 12b Target velocity profiles.

The dynamic pressure Q is defined as

$$Q = 0.5\rho v_t^2 \tag{20}$$

with the total target velocity v_t obtained from

$$v_t = \sqrt{v_{tx}^2 + v_{th}^2} \tag{21}$$

Because the acceleration equations are in a fixed or inertial frame, they can be integrated directly to yield velocity and position.

B. Target Trajectories

Two tactical ballistic target flight trajectories with flight-path angles of 45 and 60 deg (Fig. 12a) and terminal speeds of 5–6 Mach (Fig. 12b) are used in the simulation studies. Our focus is put on the interval where the target is diving.

C. Mission Objective

The final miss distance should be within 5 m whenever the fuzzy logic guidance or conventional guidance with constant gains is used.

D. Initial Conditions

The initial conditions were as follows. For the missile, $m =$ 600 kg, $T = 100{,}000$ N, $x = 0$ m, $h = 0$ m, $\alpha = 10$ deg, and $\gamma =$ 60 deg. For the target, $v_t = 1800$ m/s; $\gamma_t = 45$ deg; and $(x_t, h_t) =$ (30, 29) km, (30, 30) km, and (30, 31.5) km. Nominal physical and aerodynamic parameters for the missile and target are given, respectively, in Tables 1 and 2.

Table 1 Nominal physical and aerodynamic parameters of missile

Parameters	Values
Reference area	$s = 0.086\ \mathrm{m}^2$
Initial weight	$m = 600$ kg
Final weight	$m = 350$ kg
Max thrust	$T_{\max} = 100{,}000$ N
Min thrust	$T_{\min} = 0$ N
Zero drag coefficient	$C_{D0} = 0.45 - (0.04/3)$ Mach
Lift coefficient	$C_{L\alpha} = \partial C_L / \partial \alpha = 2.93 + 0.34008\,\mathrm{Mach} + 0.2615\,\mathrm{Mach}^2 + 0.01085\,\mathrm{Mach}^3$
Induced drag coefficient	$\mu = 0.053$

Table 2 Nominal physical and aerodynamic parameters of target

Parameters	Values
Reference area	$S_{\mathrm{tref}} = 0.176\ \mathrm{m}^2$
Zero lift drag coefficient	$C_{tD0} = 1.25$
Weight	$W_t = 1816$ kg

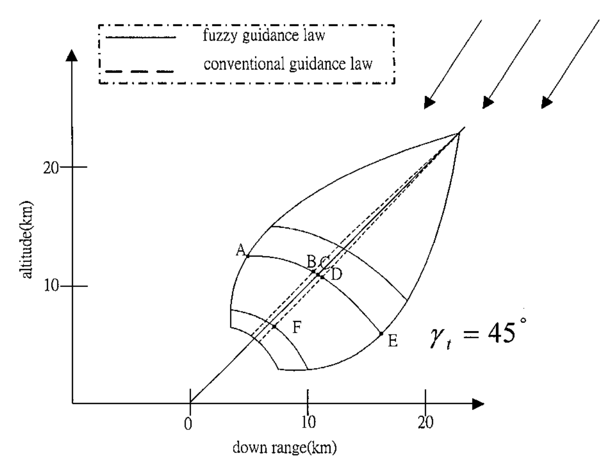

Fig. 13 Defensible volume for target with reentry angle of 45 deg.

E. Defensible Volume

We divide the discussion into two parts: 1) Conventional and fuzzy logic guidance designs are considered, and the ballistic target has a 45-deg reentry angle and 1800 m/s velocity, but the altitude and down range are shifted arbitrarily. 2) Only the fuzzy logic guidance is considered, and the target is assumed to have reentry angles of 45 and 60 deg.

1) Figure 13 shows the defensible region with the missile guided by various guidance laws. Simulation results show that the defensible region obtained by the fuzzy logic guidance is far wider than that obtained by the conventional design. Because the air density decreases with the increasing altitude, the missile aerodynamic maneuverability decreases at higher altitude. On the other hand, the total flight time is not long enough for the missile to build up speed if it engages the target at a lower altitude. It is difficult for the missile to engage targets at these regions. It is also found that the best interception point occurs at altitude 10 km and downrange 17 km. The engagement trajectories corresponding to the points A–E in Fig. 13 are shown in Fig. 14.

2) Figures 13 and 15 show the defensible regions with respect to the target having the same velocity, 1800 m/s, but with different reentry angles. It can be found that the defensible regions decrease as the reentry angles of the target increase. The target with larger reentry angles yields higher vertical speed. For that case, the available time for the missile to successfully engage the target is relatively short. As a result, the defensible region is comparatively smaller.

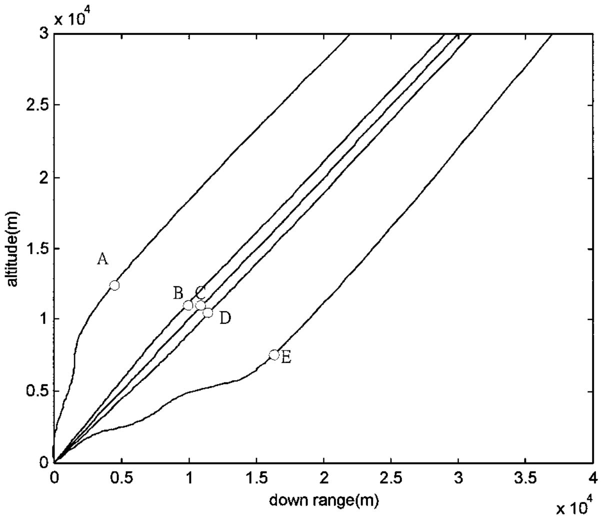

Fig. 14 Trajectories of engagement with fuzzy logic guidance.

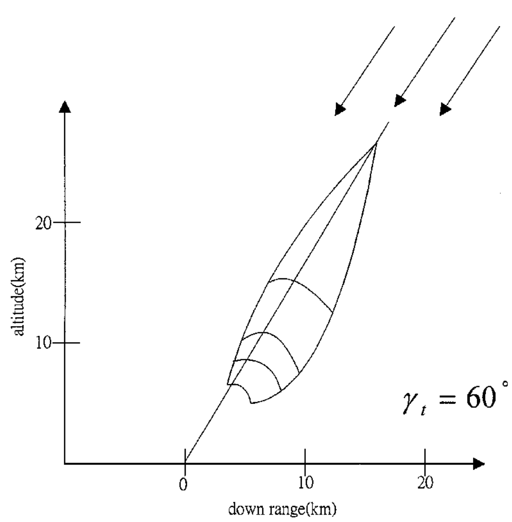

Fig. 15 Defensible volume for target with reentry angle of 60 deg.

F. Tracking Error and Control Effort

Figures 16 and 17 show, respectively, the predicted velocity error angle, the heading error angle, and the angle of attack for the conventional and fuzzy logic guidance designs with the engagements corresponding to the points B and D in Fig. 14. From the tracking error histories it can be found that the conventional guidance converges slowly, and the error is larger than that of the fuzzy design. Therefore, the overall control energy consumption for the latter should be smaller than that of the former.

From Figs. 16a and 16b and 17a and 17b, it is found that the velocity error angle converges faster than the heading error angle. This can be explained as follows. In midcourse phase, the guidance is expected to guide the missile nearly head-on with the target before it enters terminal phase. The head-on condition is mainly determined by the velocity error angle. In midcourse phase, the velocity error angle is placed with a higher weight than the heading error angle in rule design. Therefore, convergence of the former should be quicker than the latter.

In terminal phase, a modification of the traditional PD-type fuzzy rule table[11] is used. From the transient response of the heading error angle, we can find that the PD rule table works well for the missile terminal guidance.

From Figs. 16c and 17c, it can be observed that control effort in the constant gain design is larger than that of the fuzzy logic design. This implies that the fuzzy logic guidance has the potential to yield greater terminal speed and to provide higher maneuverability. The result shows that considerable energy consumption savings can be obtained if expertise has been appropriately incorporated into the fuzzy rule.

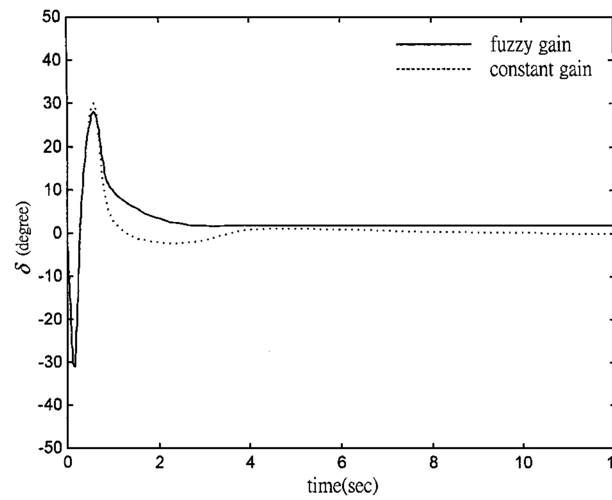

Fig. 16a Velocity error angle.

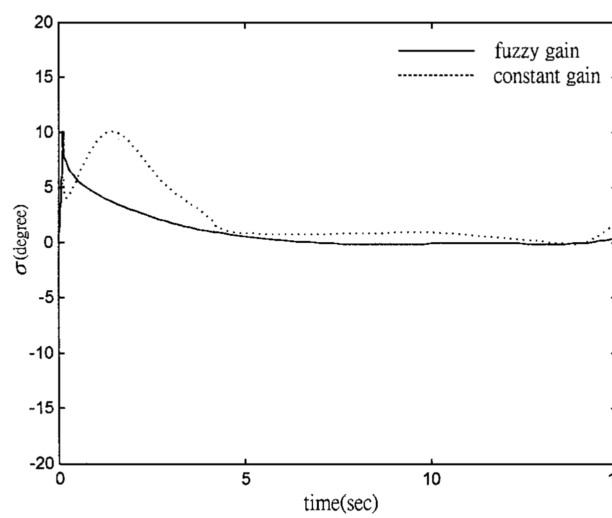

Fig. 16b Heading error angle.

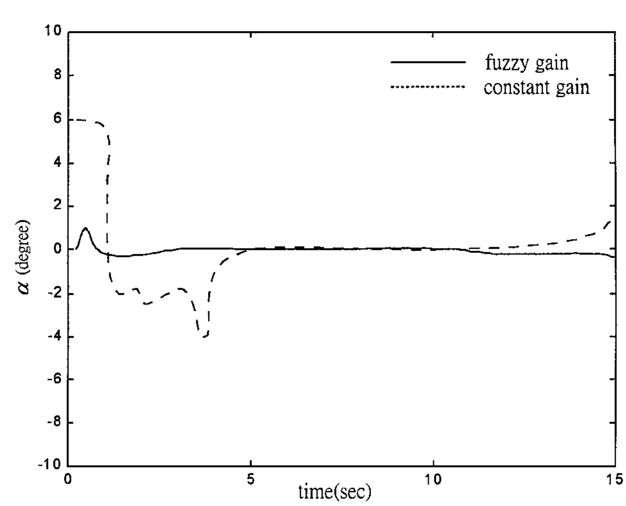

Fig. 16c Angle of attack, corresponding to point B in Fig. 14.

G. Robustness

Robustness of the fuzzy logic design to variations in aerodynamic coefficients is discussed. We adjust the lift coefficient C_L to 50 and 80% of the nominal value, and the drag coefficient C_D to 120% of the nominal value. Velocity and heading error angles with aerodynamic coefficient variations are shown, respectively, in Figs. 18a and 18b. These error responses correspond to the point F in Fig. 13. Figures 18c and 18d are, respectively, the heading error responses corresponding to the points A and E in Fig. 13. With the same extent of aerodynamic variations, it is observed that robustness of the fuzzy

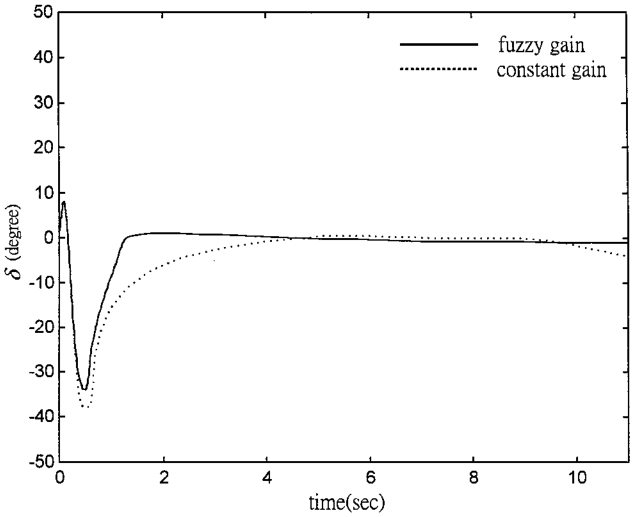

Fig. 17a Velocity error angle.

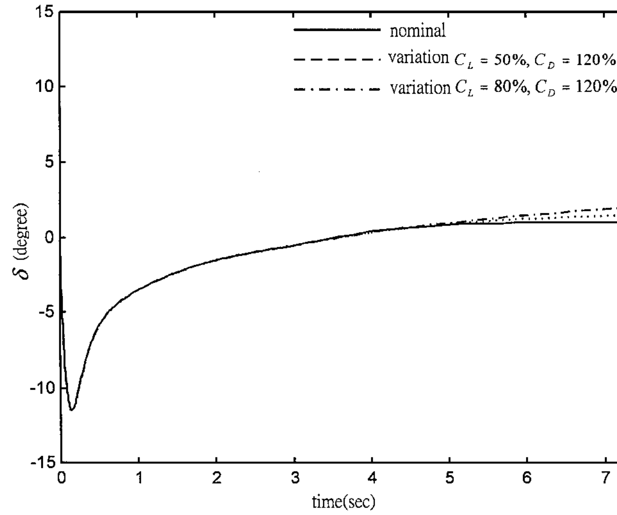

Fig. 18a Velocity error angle to aerodynamic variation (corresponding to point F).

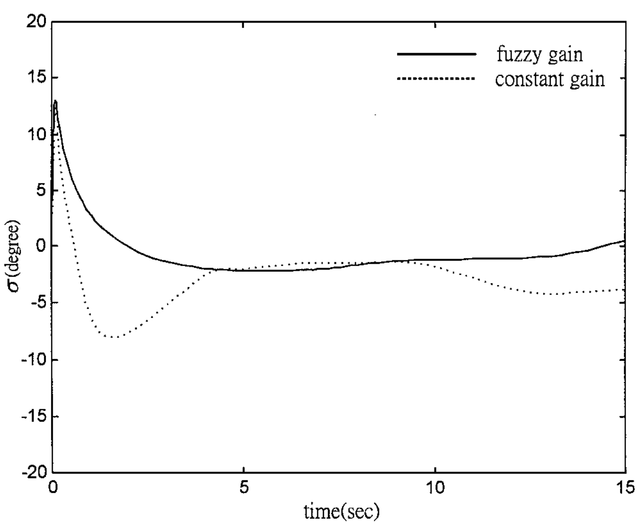

Fig. 17b Heading error angle.

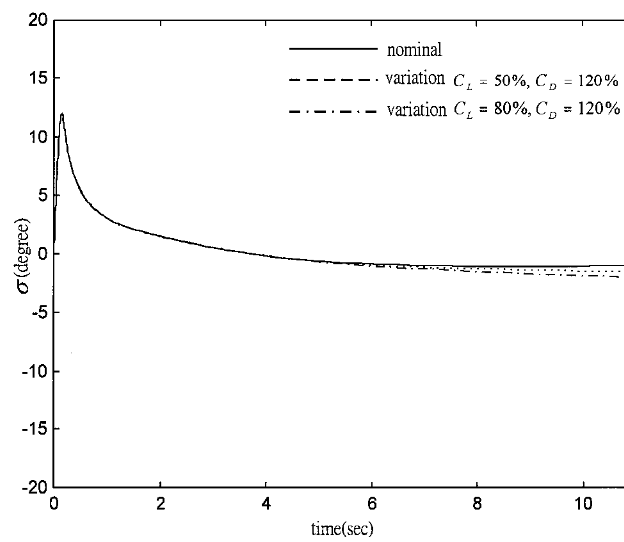

Fig. 18b Heading error angle to aerodynamic variation (corresponding to point F).

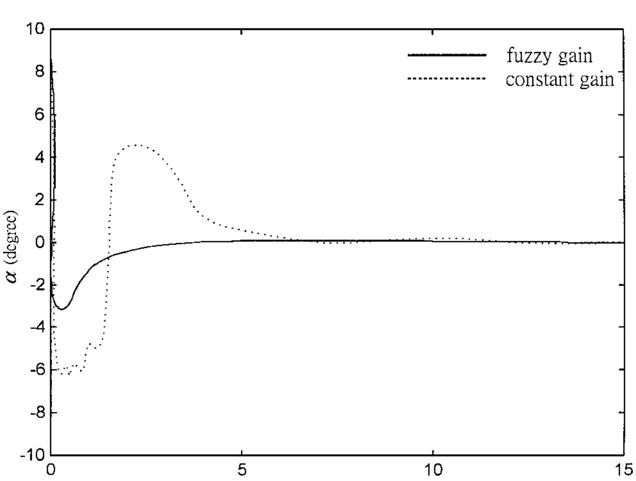

Fig. 17c Angle of attack, corresponding to point D in Fig. 14.

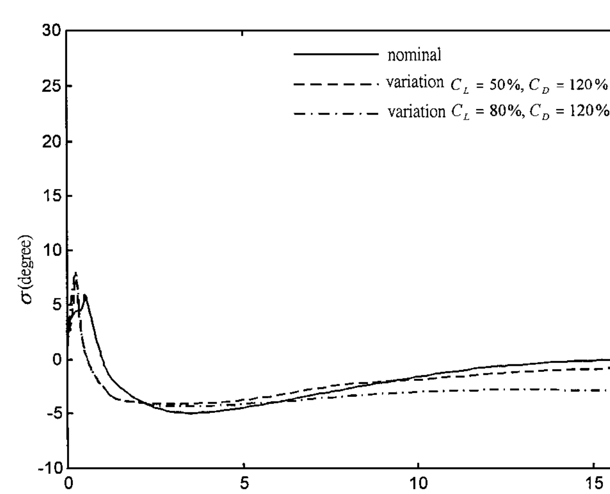

Fig. 18c Velocity error angle to aerodynamic variation (corresponding to point A).

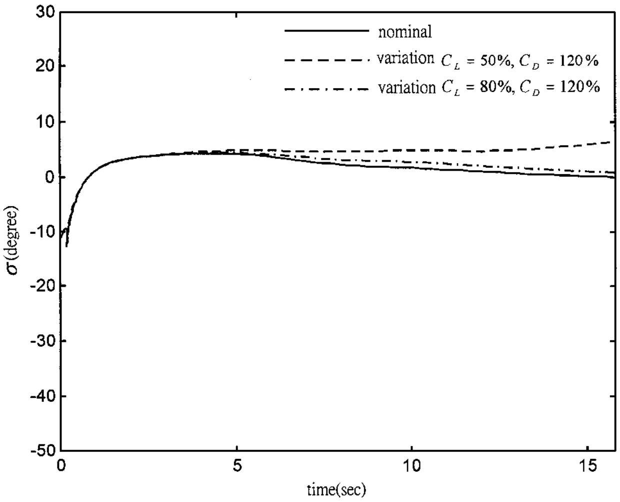

Fig. 18d Velocity error angle to aerodynamic variation (corresponding to point E).

guidance design degrades in higher engagement altitudes. This is because the missile's maneuverability decreases gradually in higher altitudes and, thus, weakens the tracking capability. However, this design still offers satisfactory performance when in the presence of large aerodynamic variations.

VI. Conclusion

A fuzzy logic-based guidance scheme is proposed to guide a missile engaging the incoming target with very high speed. The fuzzy midcourse guidance law is designed to increase the terminal speed at lock-on point and to provide a better counterattack condition, that is, the target aspect angle at lock-on is near head-on. The fuzzy terminal guidance law is designed to accelerate the tracking response and to avoid overshooting the missile response. The guidance law is easily implemented and is less sensitive to aerodynamic changes.

Compared to the conventional explicit guidance design in which control gains are the solutions satisfying the boundary conditions, the proposed design uses less control energy and, in addition, yields better miss distance performance and wider defensible volumes. Simulation results show that the proposed guidance scheme offers great potential for the development of high-performance missiles.

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